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Comparison and contrast in soil depth evolution for steady-state and stochastic erosion processes: Possible implications for landslide prediction

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Key points:

- 1) Steady-state soil depth is proportional to the infiltration, but inversely proportional to the erosion rate, when erosion is gradual.
- 2) A quasi-universal prediction of soil depth as a function of slope angle is generated for gradual erosion and landsliding.
- 3) While mean slope depths for gradual erosion and landsliding are similar, for the same total erosion rate, greater depths can be achieved in the latter.

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Abstract

The importance of gradual erosion relative to landsliding depends predominantly on the slope angle. One factor of critical influence in landsliding along with slope angle and slope shape is the soil depth. Understanding soil depth development on steep topography is fundamental for understanding and predicting the occurrence of landsliding at threshold landscapes. We develop a model to predict soil depth that addresses both threshold and gradual processes.

If erosion is a gradual process, soil depth increases until the soil production rate no longer exceeds the erosion rate, and steady-state is reached. The predicted soil depth (x) is proportional to the ratio of the infiltration to the erosion rate. Identifying a predictive result for erosion as a function of slope angle (S) allows a test of both the erosion and soil production models with field observations. The same theoretical approach to soil production should be applicable when the principal erosion process is shallow landsliding. After landslides, soil recovery initially follows our predicted power-law increase in time, though with increasing time background erosion processes become important. At a time equal to a landslide recurrence interval, the soil depth can exceed the steady-state depth by as much as a factor 2. By comparing predicted and observed $x(S)$ results, we show that the accessed result for erosion as a function of slope angle is accurate. Soils deeper than the depth predicted at the landslide recurrence interval are beyond the stability limit. This result suggests an important practical relevance of the new soil production function.

Keywords: soil production function, landslides, threshold slope, percolation theory

1. Introduction

Johnson & Schaetzl (2015) review two prominent competing frameworks to understand soil production that can be traced either to Darwin (1881) (bioturbation), or to Dokuchaev (1948) (soil formation factors, including e.g., climate). The former can be applied to understand particle size zonation, while the latter is amenable to derivation in terms of the limits introduced by chemical weathering (Yu *et al.*, 2017). In the latter theoretical understanding, a primary importance is assigned to vertical water fluxes in promoting the chemical weathering by bringing in weathering reagents and transporting out weathering products. Water pathways in underlying bedrock are generally recognized to be fractures, but the dominant mechanism in propagating such fractures has not been identified, as both the roles of tree roots (Gilbert, 1877; Pawlik *et al.*, 2016) and chemical weathering (Eppes & Keanini, 2017) have been established. In any case, quantifying the production and erosion of soil is key in a wide range of agricultural (Blanco and Lal, 2010; Montgomery, 2007a), cultural history (Montgomery, 2007b), geomorphological (Gilbert, 1877; Heimsath *et al.*, 1997), and climatic studies (Kump *et al.*, 2000), including elemental cycling (Hartmann *et al.*, 2014).

The constraint placed on chemical weathering from solute transport is a phenomenon that is receiving increased attention as possibly the most important limitation on soil production (Burke *et al.*, 2006,2009; Dixon *et al.*, 2009ab; Anderson & Anderson, 2010; Maher, 2010; Egli *et al.*, 2014). Hunt *et al.* (2016) recently developed the theoretical framework for modelling soil production and chemical weathering based on solute transport limitations. It has, in common with other commonly used soil production models (e.g., the exponential phenomenology of Heimsath *et al.*, 1997), the feature that soil production diminishes with increasing depth. However, it differs from the exponential phenomenology in that its spatial dependence is a power-law, making the temporal evolution also a power-law, in

accord with observed chemical weathering rates (e.g., White & Brantley, 2003). Moreover, the parameters of the function have direct physical origins traceable to the values of vertical water fluxes and typical particle sizes (e.g., Yu *et al.*, 2017). Further, it is in accord with a century of the theory of soil development (Yu *et al.*, 2017), from Dokuchaev (1948) onwards (translation 1948, original from 1883).

The derived soil production rate is proportional to the infiltration (water) flux, I . The power-law function of time and proportionality to infiltration have direct implications for the question of whether landscapes are in steady-state; there is neither implicit assumption of a maximum, or limiting soil depth, nor is there an identification of a maximum in soil production in the limit of vanishing soil thickness, although the theoretical bounds on the applicability of the theory place practical limitations in soil production. Further, the result for the steady-state soil thickness is proportional to $(I/D)^{1.15}$, with D the denudation rate. Such a power-law can be contrasted with other formulations available in the literature (Heimsath *et al.*, 1997, 1999; Roering, 2008), which, due to the assumed exponential dependence of soil production on depth, generate logarithmic functions of depth on soil production and thus erosion. One reason given to provide understanding for an exponential form for the depth dependence of soil production is the concept of soil development through bioturbation (Huntly & Inouye, 1988; Yoo *et al.*, 2005). Bioturbation models can be applied effectively to explain layering in particle sizes (Johnson & Schaetzl, 2015 and references therein).

The emphasis here is on predicting specific values of the soil depth as a function of erosion rate based on a local condition of steady-state. It is not on the specific hillslope form developed or on the surface transport properties of the system. We will take the erosion rate, for example, as an input parameter without constraining its values from point-to-point by the conservation of surface sediment flux. Heimsath *et al.* (1999) argue that use of the diffusion equation for surface sediment transport leads to steady-state values of the soil production rate which are proportional to the

negative of the landscape curvature, i.e., higher on ridgetops where soil is eroded, and lower in hollows. Here, a result with practical similarities is obtained from a soil production function which is linearly proportional to the infiltration water flux. Since, when not limited by bedrock hydraulic conductivity values, infiltration will typically be higher in hollows and lower on ridgetops (Dunne et al., 1991), soil depths will adjust to be lower on ridgetops in order to tend towards a spatially uniform soil production and denudation rate. This argument depends on surface water routing, thus on regolith, or soil, hydraulic conductivity values compared with rainfall and transpiration rates, although the bedrock hydraulic conductivity on ridgetops may be higher than in valleys (St. Clair et al. 2015).

In summary, the curvature of a hill promotes a similar spatial variability in soil depth in both the exponential model and in ours; directly in our case, due to the variation in infiltration, while in the case of Heimsath et al. (1999) it is a result of the adjustment of the production rate to the erosion rate under the assumption of steady-state evolution at the landscape level. This distinction appears to make our results for soil production and soil depth relevant to different locations in a given watershed, as well as to different watersheds with distinct climates. The strong climatic dependence of soil production function parameters, e.g., infiltration rate, and the power-law dependence of the soil depth on erosion rate (rather than the more typical logarithmic phenomenology) also suggest that there may be important implications for the evolution of steep topography with strongly variable precipitation and erosion rates.

Studies of steep topography are of continuing interest in geomorphology. The morphologic characteristics of landscapes reflect the complex feedback between tectonics and climate-driven processes in sculpting the topography. Tectonically active landscapes are often exposed to natural hazards, such as landslides, debris flows, floods and earthquakes. These phenomena typify threshold processes that generate significant impacts on topography. Unresolved questions about threshold

landscapes range from the conceptual (DiBiase *et al.*, 2012), such as whether such landscapes respond obviously and meaningfully to changes in erosion rates, to the practical (e.g., Montgomery & Brandon, 2002), such as how to predict such erosion rates. What constitutes steady-state in a threshold landscape? Several worldwide studies demonstrate an equivalence between soil production and soil erosion. In a threshold landscape, such an equivalence can only reflect a spatial, or long-term average. The relevance of steady-state landscapes has been called into question (Phillips, 2010; Yu & Hunt, 2017a), even though an equivalence of soil production and soil erosion rates is often assumed. Nevertheless, the investigation of Yu and Hunt (2017a) indicated that, while slowly evolving landscapes in arid continental interiors were unlikely to be in steady-state, tectonically active regions were much more likely to conform to steady-state conditions, at least if erosion processes were largely gradual. The same conclusion was reached in a study of Braun *et al.* (2016).

Many studies have addressed quantitative understanding of the relationship between erosion rate and topographic elements including hillslope gradient, topographic relief, hilltop curvature, and drainage density (e.g., Gilbert, 1877; Ahnert, 1970; Montgomery & Brandon, 2002; Binnie *et al.*, 2007; Roering, 2008; DiBiase *et al.*, 2010; Hurst *et al.*, 2012; DiBiase *et al.*, 2012). General conclusions, however, have been slow to develop.

Ahnert (1970) reported a linear relation between erosion rate and mean local relief at mid-latitude drainage basins. However, several studies have demonstrated that the linear relationship breaks down as the mean slope increases and approaches a threshold angle of stability S_c (e.g., Carson & Petley, 1970; Schmidt & Montgomery, 1995; Ouimet *et al.*, 2009; Montgomery and Brandon, 2002; Binnie *et al.*, 2007; DiBiase *et al.*, 2010), at which downslope sediment fluxes can become infinite (Roering *et al.*, 2007). In this case, sediment flux switches from creep-related process to mass wasting (DiBiase *et al.*, 2012), and landslides can occur, such that hillslope lowering prevents the hillslope from becoming steeper than S_c , and erosion rate and

topographic relief become decoupled (Schmidt & Montgomery, 1995; Burbank *et al.*, 1996; Montgomery, 2001; Montgomery & Brandon, 2002). The fundamental characteristics of the empirical erosion formula given by these last authors incorporates effects from both gradual erosion and landsliding and will be applied in the modeling here.

Accurate modelling of landsliding processes is relevant not only to a general understanding of geomorphology, but also to prediction and diagnosis of landsliding hazards. According to Claessens *et al.* (2007) “High annual rainfall, steep slopes, deforestation, high weathering rates and slope material with a low shear strength or high clay content are considered the preparatory causal factors for mass movements.” In order to make predictions of when such landsliding may be imminent, one must also take into account the moisture history of slopes, modelling the spatial distribution of moisture, pore pressure, and soil strength (Dietrich *et al.*, 1995), addressing Coulomb failure and friction forces (Dietrich *et al.*, 2007), and many combinations thereof (Claessens *et al.*, 2007).

Nevertheless, soil depth is often considered (Okimura, 1987; Iida, 1999) to be the most important parameter for predicting a general risk of shallow landsliding. Thus, understanding soil depth development on steep topography may be useful for understanding and predicting the frequency of occurrence of landsliding in threshold landscapes. But modeling threshold processes, let alone landscapes, holds additional theoretical and practical challenges.

Up to the present, our theoretical development has addressed only the evolution of soil depth for gradual erosion processes. In order to broaden the investigation, we now also investigate theoretically mean soil depths between landslides in threshold landscapes. Here it is shown that, for our power-law dependence on soil depth of the soil production function (SPR), the mean soil depth is given by an analytical result almost identical to that for the steady-state soil depth for gradual erosion. Thus, for the same erosion rate, the predicted mean soil depth is scarcely dependent on

whether the erosion is by gradual or threshold processes. Therefore, we propose to predict mean soil depths in both landslide-dominated landscapes and landscapes dominated by gradual erosion processes such as soil creep, as long as one has specific evidence for the magnitude of the relevant erosion processes. In any case, soil depth clearly varies with hillslope gradient. Thus one requires either suitable measurements of erosion rates, or an accurate model of erosion rate with hillslope gradient. For the latter situation we adopted the phenomenology proposed by Montgomery & Brandon (2002) (MB).

Influences of climate, parent material, relief, and time also vary from place to place. Consequently, the soil production function (SPR) must be evaluated at individual sites using either existing parameters, or alternatively, the best guidance for generating such parameters from published information.

Using climate, soil texture, and erosion data from the San Gabriel Mountains (SGM) in California, USA, we parametrize the MB slope-dependent erosion rate function and then compare predictions of the slope-angle dependence of soil depth and soil production with field results for soil production functions and soil depth. Such investigations are then repeated at other sites, even though typically less information was available for verification purposes. For the SGM, these tests also utilize exponential SPR formulations for comparison. We also investigate the validity of our proposed SPR and its predictions for soil depths as a function of time on slopes recovering from landslides.

Should the proposed SPR be assessed to be useful, its implications for regional and local variability in soil production may help solve related questions in geomorphology, such as whether soil transport is linear (Heimsath et al. 2005), or how to predict regional denudation rates, both under past climates and for future scenarios.

Finally, we discuss the potential relevance of our prediction for the SPR to landslide risk assessment. Here, the focus is not to evaluate spatial variability in

stochastic triggers, such as rainstorms, fires, or land-use changes, in development of real-time landslide risk assessment. But, if it proves possible to generate guidance on which slopes may exhibit elevated risk for landsliding based on their depths and the slope angle, this will be a useful advance in itself.

2. General Model

We describe initially some theoretical extremes before combining them into a single, more generally applicable, model. The first is a soil production model without erosion. Then erosion is added, initially as an exclusively gradual process, later as a stochastic process representing a catastrophic (threshold) phenomenon of total soil loss by landsliding. In these tests, the analytical result of Montgomery and Brandon (2002) is applied. In each case, the soil production model remains the same, however, and we start with that input.

2.1. Soil Production Models

The present discussion will focus on the percolation model applied here as well as contrast it with a related power-law model of soil production. It will also provide sufficient detail to understand subtleties in the exponential soil production model.

Percolation Model

Our model for soil formation derives from solute transport theory in porous media (Hunt & Skinner, 2008) developed from percolation theory. For a recent review on applying percolation theory, PT, to solute transport, chemical weathering and soil formation, see Hunt & Sahimi (2017). Within the framework of PT, solute velocity diminishes with increasing solute transport distance in porous media, and the scaling of travel time (t) on travel distance (x) is determined by the fractional dimensionality of

the percolation backbone (D_b), $t \propto x^{D_b}$. Here, $D_b = 1.87$, for 3D media under conditions of full saturation (Sheppard *et al.*, 1999), and is nearly the same, 1.861 under wetting conditions. If most downward solute transport occurs during wetting or under saturated conditions, then it is reasonable to use $D_b = 1.87$ generally.

This scaling result from PT is argued (Sahimi, 1994) to be generally applicable to solute transport in disordered porous media on account of the tendency for flow to be concentrated along tortuous paths formed from the least resistive elements of the medium. Such paths can, in principle, be identified using the critical percolation probability, and their tortuosity is described by a fractal dimension, rather than a constant factor. Because chemical weathering in the field is indicated to be solute transport-limited (Yu & Hunt 2017b), as anticipated by Maher (2010), close correlation of chemical weathering rates with soil production rates (Hunt & Ghanbarian, 2016; Egli *et al.*, 2012, 2014) makes it possible to relate soil transport distances and soil depths. Thus, in the absence of erosion,

$$x = x_0 \left(\frac{t}{t_0} \right)^{1/D_b} = x_0 \left(\frac{t}{t_0} \right)^{1/1.87} \quad \text{Eq. (1)}$$

describes the evolution of soil depth x as a function of time t by using the scaling properties of solute transport in disordered networks. In Eq. (1), the length scale, x_0 , and time scale, t_0 , are required by dimensional analysis, but these constants have physical relevance, and are connected to the fundamental structure of a porous medium as a network. Consequently, x_0 is identified (Yu *et al.*, 2017) as a typical particle size, for which we substitute d_{50} , the median value. t_0 is the typical pore crossing time under fluid flow at a characteristic pore-scale flow rate, v_0 .

By taking the derivative of soil depth, one can derive the equation for soil production (equivalent to a solute, or weathering front, velocity) (R_s),

$$\frac{dx}{dt} = R_s = \frac{1}{1.87} \frac{x_0}{t_0} \left(\frac{t}{t_0} \right)^{-0.47} = \frac{1}{1.87} \frac{I}{\phi} \left(\frac{x}{x_0} \right)^{-0.87} \quad Eq. (2)$$

Here, $\frac{I}{\phi} \equiv x_0/t_0$ is the pore-scale vertical flow rate at the bottom of the soil column (Yu et al., 2017), i.e., net (deep) infiltration rate I with porosity ϕ , and $I = \text{Precipitation } (P) - \text{Actual Evapotranspiration } (AET) + \text{run-on} - \text{run-off}$ by overland flow (Hunt & Ghanbarian, 2016). It should be kept in mind that even though the pore-scale constants in Eq. (2) relate geologic time and space scales to pore-scale processes, Eq. (2) only becomes limiting for soil formation rates at time and spatial scales for which the product of Eq. (2) and the molar density of the principle reaction products is less than the initial reaction rate from reaction kinetics (Yu & Hunt, 2017b). Where this occurs varies, ranging apparently from length scales less than a millimeter (Maher, 2010; Yu & Hunt, 2017b; Egli et al. 2018) up to centimeters (Egli et al. 2018).

Exponential Model

The power-law soil production function in Eq. (2) contrasts with the exponential model of Heimsath et al. (1997; 2012), where R (which corresponds to dx/dt in Eq. (2)) = $R_0 \exp(-x/x_s)$. Here x_s is typically about 0.5m, x is the thickness of soil, and R_0 is a maximum soil production rate, obtained from extrapolation of experimental soil production data to zero soil thickness. In usual applications, steady state conditions are invoked to equate R with a denudation rate, D , yielding a soil depth. Inversion of the resulting exponential relationship $D = R_0 \exp(-x/x_s)$ yields,

$$x = x_s \ln \left(\frac{R_0}{D} \right) \quad Eq. (3)$$

Except when D is very near R_0 , this logarithmic relationship is insensitive to R_0 and D , and it is highly unusual for x to greatly exceed x_s , in accord with the global tendency for soil depths (Hillel, 2005) to be on the order of 1m.

Related Model (Braun et al. 2016)

The model of Braun et al. (2016) yields chemical weathering and soil formation proportional to flow rates, as well as to soil depth to a power of time similar to ours. The work couples hillslope evolution models with water table evolution in time, but uses zero hydraulic conductivity for bedrock and assumes initial conditions of full saturation over the entire slope. Their result for regolith thickness, x , at the bottom of a hillslope is $x = ((LP/K_h S)^2 + 2FPLt)^{1/2}$, with L slope length, K_h (m yr^{-1}) the hydraulic conductivity, F a dimensionless parameter proportional to the weathering reaction rate, P the precipitation (m yr^{-1}) S (dimensionless) the slope gradient and t (yr) the time elapsed since the entire hillslope becomes unsaturated. In contrast, field experiments exhibit a stronger dependence of regolith thickness on reaction rate at shorter than at longer times (Hunt & Ghanbarian, 2016; Egli et al., 2018), while a regolith thickness inversely proportional to K_h at early times is not consistent with the result that higher infiltration rates produce higher soil formation rates.

Harman et al., (2017) argue that an inconsistency in the approach leads to a concentration of weathering products at the bottom of the slope much larger than the equilibrium value. Harman et al. (2017) attribute the discrepancy to inconsistent treatment of the dimensionality of flow. However, Braun et al. (2017) assert that any discrepancy is removed if one does not assume that the reaction products in solution are in equilibrium, similar to our understanding.

The square-root dependence of soil depth on duration of unsaturated conditions recalls Philip (1957; 1967) infiltration theory for the infiltration depth; preserving even the dependence on the square root of K_h (Hunt et al., 2017). The inference of the importance of unsaturated conditions, however, precludes using the Braun et al. (2016) model for weathering experiments in (1D) columns under saturated conditions (Salehikhoo et al., 2013; White & Brantley, 2003), but which produce the same proportionality to flow rate and the same time-dependence of weathering rates as do

field experiments. In contrast, our soil formation and chemical weathering model is applicable in both lab (Hunt *et al.*, 2015) and field settings (Hunt *et al.*, 2014; Hunt & Ghanbarian, 2017), and under either saturated or unsaturated conditions.

2.2. Causes of Soil or Regolith Loss

The mantle of soil can form in unconsolidated material, such as alluvium, landslide deposits, mining tailings, etc., or from bedrock. Erosion of such unconsolidated material is occurring constantly through physical and chemical processes. For example, chemical weathering products are partly soluble and for some substrates, such as carbonates, removal of mass in solution is particularly important. Thus, it is advisable to consider both chemical and physical agents for loss of mass from the surface, and define a total denudation rate, $D(t)$, which can, in principle, depend on time.

2.3 Gradual Erosion Treatment

When a time-dependent denudation rate, $D(t)$, is included, one can now obtain the net formation rate of soil, equal to the time derivative of its depth,

$$\frac{dx}{dt} = R_s - D(t) = \frac{1}{1.87} \frac{I}{\phi} \left(\frac{x}{x_0} \right)^{-0.87} - D(t) \quad \text{Eq. (4)}$$

Physical soil erosion is chiefly accomplished through advective processes such as overland flow and rainsplash (Anderson & Anderson, 2010). The erosion rate at a given site relates to various factors including climate, particle size and local variations in slope and curvature, as well as total relief (Montgomery & Brandon, 2002; von Blanckenburg 2005). It can vary over about 4 orders of magnitude, from less than a meter per million years in the interior of Australia (Bierman & Nichols, 2004) or the

Atacama Desert (Owen *et al.*, 2010) to over 1000 meters per million years in the Himalaya (Bierman & Nichols, 2004) or New Zealand Alps (Larsen *et al.*, 2014). In the simplest case, $D(t)$ is treated as time-independent, D , although clearly such climate change as occurred at the beginning of the Holocene produced detectable changes in landscape characteristics in many locations worldwide.

D is often broken down into slope-independent, E_0 and slope-dependent terms, KS , where K is a constant of proportionality, and S is the slope angle. E_0 is then usually conceptualized as a loss of soil volume due to the removal of soil products by chemical weathering, which may also be time-dependent, since, as long as steady-state is not reached, chemical weathering and soil production are time-dependent. The term with linear dependence on slope is consistent with a steady-state solution of topography where soil production is proportional to the (negative) of the landscape curvature (e.g., Heimsath *et al.*, 1999). This result is then coupled with the inference (Heimsath *et al.*, 1999) that soil production on the top of hills is greater than that at the bottom, with an associated lateral transport of soil from the top to the bottom. For steeper slopes and threshold topographies, more complicated forms of erosion rates from physical processes, including a dependence on depth, may be considered (Roering, 2008). Whatever the particular form of the slope dependence, whenever the long-term erosion rate can be considered time-independent ($D(t) = D$), Eq. (4) becomes,

$$\frac{dx}{dt} = R_s - D = \frac{1}{1.87} \frac{l}{\phi} \left(\frac{x}{x_0} \right)^{1-D_b} - D \quad \text{Eq. (5)}$$

As long as $R_s \neq D$, $\frac{dx}{dt} \neq 0$, i.e., the soil thickness changes. If initially, $R_s > D$ ($R_s < D$) the soil progressively deepens (thins) until $R_s = D$. At this point $\frac{dx}{dt} = 0$ and steady-state is reached. Setting $\frac{dx}{dt} = 0$ yields for the steady-state soil depth, x_{ss} ,

$$x_{ss} = x_0 \left(\frac{1}{1.87} \frac{I}{\phi D} \right)^{1/(D_b-1)} = x_0 \left(\frac{1}{1.87} \frac{I}{\phi D} \right)^{1.15} \quad Eq. (6)$$

Here the combination of exponents ($1/(D_b - 1)$) yields the power 1.15. Thus, with the knowledge of relations between a steady-state denudation rate, $D(S)$ as a function of mean slope angle, S , plus reliable estimations of the parameters x_0 and I , Eq. (6) can be used to predict $x_{ss}(S)$.

Further, if erosion is purely a gradual process, soil depth at any age can be predicted from the integration of Eq. (5),

$$x = \int_0^t dt' (R_s - D) = \int_0^t \left(\frac{1}{1.87} \frac{I}{\phi} \left(\frac{x(t')}{x_0} \right)^{-0.87} - D \right) dt' \quad Eq. (7)$$

2.4. Threshold Process Modelling

Where landslides dominate the erosion, the purely gradual approach is not valid. Consider first how to address a problem in which gradual processes are, in some fashion, small enough to be negligible in order to understand the overall effects of threshold erosion processes. Although we consider this distinction in greater detail in Section 4, an initial assessment can be based on the magnitude of the contribution of landslides to the total denudation rate compared with that due to gradual processes. If the former is much larger than the latter, to first approximation, the gradual processes can be neglected. When the gradual contribution to the denudation rates is small, one can ask how the effects of an unsteady erosion rate would contribute to the mean soil development. This can be found by using the power-law soil depth equation to relate t_l , a typical time between landslides, and x_l a typical soil depth at which landslides occurs. Then $x_l/t_l = D$. Neglecting gradual erosion processes, we have

$$x_l = x_0 \left(\frac{t_l}{t_0} \right)^{\frac{1}{D_b}} = x_0 \left(\frac{x_l/D}{\phi x_0/I} \right)^{\frac{1}{D_b}} \quad Eq. (8)$$

Straightforward algebraic solution of Eq. (8) yields

$$x_l = x_o \left(\frac{I}{\phi D} \right)^{\frac{1}{D_b - 1}} = x_o \left(\frac{I}{\phi D} \right)^{1.15} \quad \text{Eq. (9)}$$

The only difference between the result of Eq. (9) and that of Eq. (6) is the absence of the factor $1.87^{-1.15}$, an effective increase in soil depth by a factor 2.05. Calling this a factor 2, Eq. (8) would imply that when the majority of erosion occurs by landsliding, the maximum soil depth that is typically reached before a landslide occurs is twice the depth that would be attained if the same erosion rate were developed in a steady process. But the apparent contrast with gradual erosion processes is reduced, if one considers the temporal mean soil depth. An average depth over the time interval $0 < t < t_l$ is $t_l^{-1} (t_l^{1.53}/(1.53))$, or $(1/1.53) t_l^{0.53}$, meaning that use of Eq. (6) to predict an average depth, instead of the final depth, would require substitution of $(0.65) t_l^{0.53}$ for $t_l^{0.53}$. A numerical factor 0.65, is not so different from the factor $1/D_b = 0.53$. We assume that the mean soil depth over similar slopes in a specific geographical region characterized by an unsteady, but mean, denudation rate D , will be the same as the temporal mean calculated above.

Thus, in both Eq. (6) and Eq. (9), the functional dependence of the soil depth on D , I , and x_o is the same. If landsliding is the dominant erosion process, for the same combination of parameters, the result from Eq. (6) gives a good estimate of the mean soil depth, while the result from Eq. (9) would give a maximum soil depth. Further, a good approximation to the mean slope depth can, for all slope angles, be found by substituting a single result for D into Eq. (6) that combines effects of both gradual and threshold erosion, even if the relative importance of landsliding increases with the slope angle. Then, where landsliding is dominant, Eq. (9) should give the maximum soil depth. However, if landsliding is not a relevant process at low slope angles, Eq. (9) will tend to overestimate maximum soil depths.

In the case that erosion is predominantly by landsliding, the exponential phenomenology, Eq. (3) yields a result that is not strictly defensible, since, until a landslide occurs, the erosion rate can be considered negligible. Since cases with mixed erosion sources are more difficult to deal with analytically, we address the case of erosion solely by landsliding. Then one has

$$R \equiv \frac{dx}{dt} = R_0 \exp\left(-\frac{x}{x_0}\right) \quad Eq. (10)$$

Eq. (10) may be integrated to generate

$$x = x_0 \ln\left(\frac{R_0 t}{x_0}\right) \quad Eq. (11)$$

If all erosion is by landsliding, one can also set x/t equal to the denudation rate. Then one finds

$$\frac{Dt}{x_0} = \ln\left(\frac{R_0 t}{x_0}\right) \quad Eq. (12)$$

Transcendental equations such as Eq. (12) cannot be solved exactly. Moreover, it turns out that for any specific value of R_0 , Eq. (12) can only be solved for a restricted range of parameters D . In particular, D cannot exceed R_0/e ; otherwise no solutions exist. For $D = R_0/e$, there is one solution, namely $Dt/x_0 = 1$, and for smaller values of D , there are two solutions. The solution with the smaller depth appears to be unphysical, as it yields an increasing soil depth with increasing erosion rate. If only the second solution is considered, it will become apparent that the distinction between the solutions of Eq. (3) and Eq. (12) is not large, at least in the range of values for which Eq. (12) has a viable solution. Thus, it may be reasonable to use Eq. (3) for the entire range of observed erosion rates. One peculiarity of this approach is,

however, apparent. For the stochastic solution in the exponential phenomenology, it may be necessary to distinguish between maximum denudation rates on steeper, landslide dominated, slopes and the maximum value of the denudation rates on shallower slopes without landsliding, since use of the smaller maximum D does not admit real solutions for soil depths on the steeper slopes. In the percolation formulation (Eq. (8)), with algebraic solution Eq. (9), there is no restriction on D , and the solution is valid at any slope angle.

2.5. Slope-Dependent Model of the Erosion Rate

The basic relationship between long-term erosion rate, D , and the mean slope, S , that we adopted to predict the dependence of soil depth on slope angle, combines a range of erosion effects. The empirical equation published in Montgomery & Brandon (2002, equation 1) (MB equation), was developed specifically for $D(S)$, in the Olympic Mountains, Washington State, USA. We reprint it here as our Eq. (13) and refer to this model as the MB model.

$$D(S) = E_0 + \frac{KS}{\left[1 - \left(\frac{S}{S_c}\right)^2\right]} \quad \text{Eq. (13)}$$

Relevant site-specific parameters that determine the value of D predicted in Eq. (13) include a constant erosion rate considered to be due to chemical weathering (E_0), a rate constant for a background physical sediment transport coefficient (K), and a threshold slope gradient (S_c). S is the slope angle. This empirical result assumes a form, KS , for the background sediment transport appropriate for smaller slopes that is proportional to the slope as suggested by Ahnert (1970) and supported by data collected by Pazzaglia & Brandon (1996). The factor in the denominator approaches zero rapidly as the critical slope, S_c , is approached, producing an erosion rate that tends to diverge, but whose effect on D at smaller slopes is reduced by squaring the

ratio of S/S_c . Eq. (13) was tested in the Olympic Mountains of the state of Washington, USA, and found to yield a reasonable dependence of erosion on slope angle. It is quite likely to have a wider applicability, though again only approximate, as it is possible to vary the parameter S_c according to observations of the maximum soil mantled slope angles in any region. The more difficult parameter to generate accurately is probably K , since its variability with climate, substrate, and vegetation cover is likely still too difficult to predict without extensive site-specific observations and modelling.

Substituting Eq. (13) into Eq. (6) yields,

$$x = x_0 \left\{ \frac{I}{1.87 * \phi * \left(E_0 + \frac{Ks}{1 - \left(\frac{S}{S_c} \right)^2} \right)} \right\}^{1.15} \quad \text{Eq. (14)}$$

In the development of Eq. (14) it was assumed that each input to the denudation rate, D , is independent of time. Eq. (14) displays high sensitivity to parameters such as I , x_0 , and the total denudation rate, D , with the uncertainty in the predicted depth equal or greater than the uncertainty in these parameters. These latter uncertainties will turn out to be as large as a factor 2.

3. Materials and Methods

3.1 Parameter Choices and Uncertainties, Testing Strategies

We compared predictions for soil depth and soil production as function of slope angle with data from sites with distinct climates. Information to generate parameters needed is incomplete, necessitating development of strategies for data collection.

Since the exponential phenomenology lacks the specific inputs into soil production, its use is more fundamentally restricted to steady-state conditions, in which soil production and soil erosion rates are assumed equal.

For testing the soil production model in the case of initial soil recovery following a landslide, it may be sufficient to generate those constants relevant to soil production, i.e., infiltration rates and particle sizes. For testing steady-state or mean soil depths, some characterization of the erosion rate is necessary. In the comparison of theory with field data, we have found a variety of sites, which allow more limited comparisons, but only one site, the SGM, for which it is possible to test independently Eq. (13) for soil erosion, as well as Eq. (14) for the soil depth as a function of erosion rate. This places the comparison with field data for the SGM at the forefront.

In the form of Eq. (14) our prediction for the soil depth as a function of slope angle has altogether 6 physical parameters: x_0 , which we have assumed (e.g., Yu *et al.*, 2017) to be a median particle size, d_{50} , I , which is given as the deep infiltration rate, ϕ , the soil porosity, E_0 , a background erosion rate due to chemical weathering, which takes into account mass lost in solution, K , the constant of proportionality in a linear dependence of erosion on a slope gradient, and S_c , a critical slope angle. For soil production data, what general guidance is available to find, e.g., I , the local infiltration rate? I depends not only on climatic variables, but also on surface water routing. Although progress is being made in specific catchments (Gu *et al.*, 2018), the only useful indication of how to take such run-on and run-off into account on our specific sites is found in global and continental estimates (e.g., Lvovitch, 1973). In particular, Lvovitch (1973) cites the global proportionality, $I = 0.23 P$, with about a 50% variability in the numerical prefactor.

Since the only information directly relevant to particle sizes that is usually available is soil texture, we had to appeal to studies that relate soil texture to median particle size (Skaggs *et al.*, 2001). The range of soil types considered is appropriate for agricultural purposes. Information for porosity was also not generally available, but

since its variability is usually limited to a factor 2, i.e., from a value of about 0.3 to 0.6, variations for the porosity were not explicitly considered, unless they could be determined from alternate sources. Soil depth from the model (Eq. (14)) is proportional to particle size, and is nearly proportional to infiltration rate (to the power of 1.15). Since soil particle size can range over many orders of magnitude, but in the field studies considered the variation in infiltration rate was typically no more than about a factor 5, our model so far reflects greater variability in prediction from soil type (or d_{50}) than infiltration rate.

How does one select values of the erosion parameters that are either 1) site-specific, or 2) approximately applicable for sites where no data is available? Overall, we found no general information to help us determine either E_0 or K . On the other hand, consider S_c . In the Olympic Mountains, soil is only sustained from shallow landsliding at slope angles below 25° (Montgomery & Brandon, 2002), even though the threshold angle is stated to be 40°. Landslide scars are also observed at the Apennine mountains site (Salciarini et al., 2006). Salciarini et al. (2006) studied slopes that ranged in overall steepness from 17.3° to 43.7°. This range cited suggests a similar critical slope angle of 40 – 45°, particularly since their data suggest a decrease in soil thickness of a factor 10 already for slope angle of 25 – 30°. These values are not greatly different from one another, meaning that the original value of 40° for the Olympic Mountains could perhaps be reasonably applied more generally, except in the SGM, where a more suitable estimate can be made. In fact, whenever no other viable options were available, we used all original erosion parameter values from the Olympic Mountains (Montgomery & Brandon, 2002).

In view of the rather extensive data and accompanying data descriptions, it was possible to generate reasonable values of all three erosion parameters for the San Gabriel Mountain (SGM; Table S1) sites investigated by Heimsath et al. (2012). Individual values for the SPR function are given at a wide range of slope angles, which can either be substituted directly into Eq. (9), or used to estimate values of the

parameters for Eq. (13). At the SGM site, it is therefore possible to address the validity of both main inputs, Eq. (9) (or Eq. (6)) and Eq. (13), to Eq. (14) separately, since it is possible to find reasonable values for l and x_0 as well, similarly to Yu et al. (2017).

3.2. Sites Used for Comparison with Landslide Recovery, Eq. (1) or Eq. (7)

We start with the simplest case, initial soil development following a landslide. Altogether three sites for comparison with theory were found. These were located in New Zealand, Japan, and Switzerland. Only for the New Zealand and Switzerland sites could detailed information for extracting parameter values be found. The data for Shimokawa's (1984) site (with granitic substrate) was republished by Iida (1999) with ages determined by dendrochronology.

Taranaki Peninsula, North Island New Zealand

The Taranaki Peninsula extends southwestward from the city of Stratford on the southwestern coast of New Zealand. The Taranaki hill country 30 km to the east of Stratford is the site of the Trustrum & de Rose (1988) landslide recovery study. The yearly mean precipitation of the Taranaki study site is 1873mm (Trustrum & de Rose, 1988). The substrate is given as a silty sandstone (Trustrum & de Rose, 1988). The Taranaki Basin, which contains the only proven hydrocarbon reserves of New Zealand, extends from the hill country along the Peninsula to offshore. Its economic relevance has led to careful characterization. A study of the evolution of porosity in the Taranaki province was conducted by Armstrong et al. (1998) in order to deduce the exhumation magnitudes and erosion rates of this region. The study was conducted on and near the Taranaki Peninsula which is located between 39° S and 39.5° S and at about 174° E. The map of the authors extends to nearly 175° E and from 38° S to 41° S. Armstrong et al. (1998) agree that the overlying rocks are a silty sandstone. The porosity values of these sandstones trend from 50% offshore (173°

E) to between 15% and 20% on the eastern margin near 175° E (Armstrong et al. 1998, Figure 6). This information was used by Armstrong et al. (1998) to infer exhumation magnitudes and corresponding denudation rates of 400 m Myr⁻¹ at the eastern margin, and even 900m Myr⁻¹ in the northeast. The southernmost onshore regions have a denudation rate of 100-200m Myr⁻¹. A second reference, McBeath (1977), states on page 124 (in reference to the Taranaki Basin sandstones), “The average porosity of gas-bearing sands penetrated by the wells is 18.8%.” McBeath also noted that early 19th Century European settlers discovered oil seeping from the surface in this region. McBeath (1977) also presents a table giving 15% as the average porosity of the petroleum-bearing rocks. We used the value 18.8%.

A median particle diameter for a silt is 15µm, while for a sand it is approximately 115µm (Skaggs et al., 2001). We take an arithmetic mean = 65 µm of 15µm and 115µm, since a silty sand should be closer to a sand than a silt. A similar result of 62µm is obtained using a geometric mean of 70% sand and 30% silt, for example. The fraction of precipitation infiltrating was again assumed to be 23% (Lvovich, 1973).

After clearing of forest by European settlers, beginning in 1840, landslides occurred rather frequently in the Taranaki hill country (Trustrum & de Rose, 1988). The historical occurrence of many of these landslides allowed their dating with uncertainty less than 1 year. The authors state, “Soil depths in landslide debris accumulation zones are generally greater than 1.5m compared with an average depth of 0.7m on hillslopes.” “Mean soil depths in the 30-40cm range have been observed on landslide scars which appear to have slipped prior to forest clearance.” “Consequently, soils are thinnest on ridges and spurs and thickest in swales and in footslopes, where soil depths up to 3m have been observed.” The authors state, “Repeated landsliding on previously failed surfaces has not been observed.” “This suggests that new ‘pasture’ soils have not yet reached a ‘steady-state’ depth, even after 80 years.”

Rossberg, Switzerland

The Rossberg site is located in the northern part of the European Alps. The geology of the Rossberg area is characterized by subalpine molasse. The alluvial deposits are part of the Lower Freshwater Molasse (Lower Freshwater Molasse) of the Oligocene. Due to its regular, geological structure with oblique, southward-dipping layers, the southern side of the Rossberg mountain has been susceptible to landslides. Historic and pre-historic landslides are known and were dated (for an overview, see Egli & Fitze, 2001; Keller, 2017). The erosion data for the Rossberg site (Table S2) were accessed from BAFU (2015) and soil characteristics from Meili (1982) and BLW (2012). Programming and calculations were done using the software R (R 3.3.2.). In order to facilitate reproducibility, the R code is made available under <https://github.com/curdon/soilDepth> (Egli et al., 2018).

3.3. San Gabriel Mountains (SGM) Parameters

Erosion-rate Parameters

Instead of using purely statistical means to create the MB parameters for the SGM, we have tried to take all the observations of the authors into account. The contribution from a background erosion to soil production is suggested to be limited to 170m Myr^{-1} or less for slopes of less than about 30° , (from page 212) “Samples collected across this more slowly eroding, convex-up part of the landscape define a robust soil-production function, with SP_{\max} equaling $170 \pm 10\text{m Myr}^{-1}$ ” while landsliding contributes up to an additional 200m Myr^{-1} at steeper slopes, for a total erosion of about 370m Myr^{-1} . “As morphology shifts from convex-up to planar, slope gradients increase and we observed the soil mantle transition from being ubiquitous to becoming increasingly patchy. We focused sampling of steep (average slope $>30^\circ$) hillslopes on smooth, locally divergent ridges away from any landslide scars, thus ensuring that our ^{10}Be concentrations represent SPRs. Importantly, we observed that

soil patches on threshold slopes, although typically thin (< 20cm) and coarse grained, are clearly produced locally and are not colluvial accumulations. SPRs from saprolite under these thin to non-existent soils are among the highest such rates ever reported, and exceed SP_{max} predicted from the low-relief soil pits by up to a factor of four, with a predicted maximum rate of $370 \pm 40\text{m Myr}^{-1}$. Heimsath et al. (2012) reported also a steepest observed slope of 45° that contained soil, meaning that the threshold slope angle is likely larger than 40° .

Other SGM Parameters

Sanford and Selnick (2012) give a precipitation range for the SGM of 51-75 cm. The middle of this range is 63 cm. The fraction corresponding to infiltration is, according to Lvovitch (1973), 23%, yielding 14.5cm yr^{-1} . Division by a typical porosity of 0.4 generates 36cm yr^{-1} . Soils in SGM, at least on hillslopes, are mainly loams (Rulli & Rosso, 2005), with median particle size ranging from 20 to $40\mu\text{m}$ (Skaggs et al., 2001). Thus, we take $30\mu\text{m}$ as a typical particle size (Yu et al., 2017). As noted, the range of reasonable values for each of the parameters is roughly a factor 2, though, e.g., adding variability in soil texture would increase this uncertainty.

3.4. Parameters for Testing Eq. (14): General Comments

Since at other sites, it was not possible to test Eq. (13) directly, we could test it only indirectly through by predicting the soil depth with Eq. (14). The near equivalence of Eq. (6) and a mean soil depth from Eq. (9) means that we can use Eq. (14) for comparison with results from arbitrary landscapes, though with uncertainty in the validity of the parameterization of the erosion rate.

In the comparison of the predictions of Eq. (14) with the slope angle, complications from uncertainty in particle size and infiltration rate are reduced in a

relative sense. Consider that Eq. (14) is a power-law prediction of soil depth, inviting logarithmic comparisons. On a bilogarithmic plot of soil depth against slope angle, the only quantity which varies strongly is D , (though l can vary with a changing vertical distance to bedrock, affected by local slope) and the net effect of changes in the two constants, l and d_{50} , is essentially to raise or lower the curve by a constant value. Rather than generating a new prediction for the combination of numerical factors appropriate to each site, as long as the fundamental parameters from the erosion model can be held constant, we can move each data set vertically, using the ratio of its estimated combined numerical coefficient to that of a known site. For the known site, we choose the SGM. For the SGM, at least, we have prior estimations (Yu & Hunt, 2017b) of l and d_{50} . At new sites we use this numerical combination as an *adjustable* parameter, in order to see if the slope dependence, at least, of the soil depth is properly accounted for. Then the optimal value of this parameter is compared with our best estimate of what available information regarding l and d_{50} would imply.

If specific evidence for a different value of the critical slope could be found, we took that qualitatively into account. Independent evidence for a background value of the erosion rate and K were not generally available. The vertical shift in experimental data is accomplished by rescaling particle diameters and infiltration rates to their values in the SGM. d_{50} 10 times as great, i.e., 300 μm , according to Eq. (14), generates soils 10 times as deep; thus at a site where the measured particle $d_{50} = 300\mu\text{m}$, we should account for that effect by dividing the experimental soil depths by 10. Effects of variability in l are treated analogously. Since estimates of the worldwide variability of mean deep infiltration rates range from 11% to 35% of annual precipitation, (Schlesinger & Jasechko, 2014; Lvovich, 1973; Peel *et al.*, 2010), as in Yu *et al.* (2017), we take the mean percentage (23%) of P to estimate l for each site. Such an estimation will introduce scatter, even if the estimated global mean is correct, since the mean value is not every value. Then, the ratio of the l value at each other site to the infiltration rate (0.36m yr^{-1}) in SGM is raised to the 1.15 power (as in Eq. (14)) to

generate a second factor to relate the steady-state soil depth at the given site to the value in the SGM. We then normalize the soil depth based on the combined effect caused by the difference in both the particle size and infiltration rate, and the values are listed in Table 1.

3.5 Sites Used for Comparison with Soil Depth as Function of Slope, Eq. (14)

Appenine Mountains

For the Apennine Mountains, central Italy (Salciarini *et al.*, 2006), soils are mainly talus and are much coarser than SGM. We estimate the particle size based on the hydraulic conductivity (10^{-5} to 10^{-2} ms^{-1}) at the site (Salciarini *et al.*, 2006) and the authors' description of the soils as talus, an especially coarse material. From information provided by Aqtesolv, USGS and USDA, (accessed in Sep. 2017), comparable hydraulic conductivities are found within the category of coarse sand, which has particle size range from $600\mu\text{m}$ to $2000\mu\text{m}$. The geometric mean of $1095\mu\text{m}$ is taken as the typical particle size.

Sterling

Soils at Sterling are mainly fine loamy and fine silty sand with percentage of sand ranging from 42% to 54% (Moore *et al.*, 1993), similar to the loamy texture in SGM, thus we applied the same particle sizes there as for SGM. When depth values are unavailable at slope close to zero, the deepest soil depths obtained (mostly in the valleys) in the field for each site are used as estimations of soil depths at zero slope. These values may be affected by landslide deposits, but they are the only possible values to choose for normalization to zero slope.

Plastic Lake

Soil texture at Plastic Lake is not mentioned by the authors. We simply kept it as 30 μm , the typical particle size of soil, which is calculated from the geometric mean of individual arithmetic means of the three principal soil particle classes, clay, silt, and sand. It is also known that silt is the middle particle size (geometric mean) class in soil classification schemes that has a mean value of 32 μm , thus a 30 μm of typical particle size would be a reasonable choice in the absence of any information regarding soil texture at Plastic Lake.

Norton & Smith

These data were simply digitized from a curve republished by Jenney (1941). Note that there was a slight upturn in the curve at the largest slope angles, which we attribute to a graphing error of the times.

4. Results and Discussion

For clarity, we first discuss model representations of the combined effects of gradual and threshold denudation processes before we address actual comparison with field measurements.

4.1. Relative Effects of Gradual and Threshold Denudation Processes

We now assess the relative contributions of gradual and catastrophic contributions to soil evolution at a point within the percolation framework. Specifically, we compare the value of t_i with the value of t_{ss} , a time to reach steady-state. Use of a power-law decay in soil production rate does not permit the usual rigorous definition of a time scale consistent with an exponential decay, but a minimum time to reach

steady-state can be found by setting the erosion-free soil depth at t_{ss} equal to the depth in steady-state. Thus we apply,

$$x = x_0 \left(\frac{t_{ss}}{t_0} \right)^{\frac{1}{D_b}} = x_0 \left(\frac{x_0/t_0}{DD_b} \right)^{\frac{1}{D_b-1}} \quad \text{Eq. (15)}$$

The solution of Eq. (15) can be represented in multiple ways. Two are given in Eq. (16).

$$t_{ss} = \frac{x_0}{DD_b} \left(\frac{I}{\emptyset DD_b} \right)^{\frac{1}{D_b-1}} = \frac{x_{ss}}{DD_b} \quad \text{Eq. (16)}$$

Here x_{ss} is the steady-state soil depth as obtained from (Eq. (6)). In the second form, t_{ss} is identifiable as proportional to the time it would take to remove the steady-state soil depth, x_{ss} , by a constant denudation rate, D , in the absence of soil production.

We can now consider the extreme cases of landslide-dominated and gradual erosion-dominated landscapes by comparing the two times, t_{ss} and t_l . Using Eq. (16), and the definition of t_l , one can see that the ratio of t_{ss}/t_l is a ratio of rates of erosion by landsliding and gradual processes. When $t_{ss} \gg t_l$, landslide erosion rates are much larger than gradual contributions, and landslides occur long before soil development approaches the steady-state value generated by the gradual erosion processes. In this case, we can use Eq. (1) for soil depth in between landslides. Since this condition leads to a well-defined invertible relationship of $x_l(t)$, we can relate typical landslide occurrence intervals to a unique soil depth, implying that we can generate a prediction for a soil depth at which a given slope becomes unstable.

In the opposite case, when $t_l \gg t_{ss}$, however, $x_l = x_{ss}$, since steady-state is virtually always reached before a landslide can occur. Further, since x_{ss} is independent of time, it is impossible to generate a unique time-depth relationship. The comparatively rare process of landsliding always occurs at the same depth in a given location (unless, e.g., climate changes), but at times which are completely

unpredictable from depth measurements, making landsliding effectively a purely random process to which our analysis lends no predictive capability. Moreover, due to the spatially variable infiltration rate alone, soil depths will also be spatially variable, even at the same slope angle, but this variability still does not permit our approach to develop an associated distinction in how close the respective slopes are to failure.

Since the importance of landsliding increases more rapidly with increasing slope angle than do gradual processes, both cases $t_l \gg t_{ss}$ and $t_{ss} \gg t_l$ may develop in the same geographic region, possibly even on the same hillslope. But, if a total denudation rate can be identified that is valid across the range of slope angles found in a given region, this relationship can be inserted into Eq. (6) to generate a prediction of soil depth, x , whether this represents a steady-state value or an average value.

In the third case, when $t_l = t_{ss}$, there is some possibility for estimating slope stability, but it is significantly degraded by the reduced sensitivity of depth to time.

Figure 1 shows a schematic diagram generated from our model with these three different scenarios. The parameters we used to generate the curves are: $I = 0.5\text{m/yr}$, $x_0 = 100\mu\text{m}$, and $\phi = 0.4$. In the first case, we show what would result for a gradual erosion rate of 100m Myr^{-1} , and no landsliding. For the second, we demonstrate the appearance of the soil development including an erosion rate by landsliding of 100m Myr^{-1} and zero gradual erosion. For the third we use 100m Myr^{-1} for both gradual and landsliding processes, producing an approximate equality of t_{ss} and t_l .

What is illustrated, in the case of pure landsliding, is the power-law increase (with power 0.53) of soil depth in time between landslides, then an abrupt loss of the entire soil column at each landslide. This choice represents a further idealization. It is not necessarily true that every landslide removes the entire soil column, particularly not at every point. In the final case, Eq. (7) must be used to generate soil deepening following landslides. For this choice of parameters, the soil depth is typically approaching a steady-state value when the landslide occurs, but the variability in

landslide recurrence intervals means that steady-state is sometimes reached and sometimes not at all.

If the present treatment of landsliding turns out to be of importance, the variability of t and x as given in Eq. (7) will constrain the particulars of any statistical approach applied to predict landsliding, since Eq. (7) is not a linear relationship.

4.2. Time-dependent Post-landslide Soil Recovery, Eq. (1) or Eq. (7)

Taranaki Peninsula, New Zealand

Of the three sites, consider first the Taranaki hill country site in New Zealand (Trustrum & de Rose, 1988). Figure 2a shows that insertion into Eq. (1) of the parameters given in the literature for porosity (18.8%), precipitation (1.873m/yr), and particle diameter (65 μm) leads to good agreement with field results using Eq. (1). The only parameter used which was not site-specific but a global mean, was the fraction of precipitation that infiltrates (between 11% and 35%, Lvovitch, 1973); these values bound the data reasonably well, although $I = 0.11 P$ appears to be too low.

We can use Eq. (5) to calculate a steady-state depth for each of the denudation rates given by Armstrong et al. (1998). For 400m Myr^{-1} , the result is 0.66m, which compares well with the stated average hillslope depth of 0.7m. The smallest onshore denudation rates given by Armstrong et al. (1998) are 200m Myr^{-1} to 100m Myr^{-1} , which generate depths of 1.47m and 3.27m, respectively. These values compare well with the two values given by Trustrum & de Rose (1988) for “accumulation regions,” 1.5m, and for footslopes and swales (3m). Interestingly, the predicted soil depth for the 900m Myr^{-1} denudation rate is 0.26m, which is somewhat smaller than the 30-40cm of soil overlying pre-settlement landslide scars, but nevertheless suggests a possible inference that the variation in denudation rates is accommodated by the prevalence or rarity of landsliding. Note that regression of the four predicted and observed steady-state soil depths, if the interpretations are legitimate, yields a slope

of 1.06 and an R^2 value of 0.992. Extrapolating Eq. (1) with the given parameters to a depth of 35cm yields a time of 312 years, while use of Eq. (10) to generate a steady-state time yields 389 years for the denudation rate of 900m Myr⁻¹. Thus, our estimates of landslide recurrence intervals generate results between 2 and 2.5 times the period since European settlement, in general accord with the conclusions of Trustrum & de Rose (1988), that this time exceeds 80 years, and probably the time since settlement.

Japan

The second site was in Japan (Shimokawa, 1984), for which no information to suggest plausible values for l or d_{50} could be accessed. If the first data point in each of the first two data sets is omitted, each conforms closely to Eq. (1), as seen in Figure 2b. The first data point from Shimokawa (1984) lies well above the trend, with the first point of Trustrum & de Rose (1988), equally below. The resulting powers, 0.54 and 0.55 (predicted value 0.53), and relatively high R^2 values of 0.89 and 0.97, suggest that, at both the Japan site and the New Zealand site, the time dependence of Eq. (1) is confirmed.

Rossberg, Switzerland

The third site is Rossberg (Swiss Alps). Using Eq. (7), the predicted soil depths match well with observed soil depths (Figure 3). The model slightly underestimates soil depth for surface ages < 1000 years and probably slightly overestimates the soil depth for older soils. The modelled trend rather displays a logarithmic function than a power-law, although R^2 (based on the regression curve of plotted values and trend) using a power-law is higher. However, Eq. (7) does predict power-law behavior only at times short enough that it can be approximated by Eq. (1).

4.3. Slope-Dependent Soil Depth, Eq. (14), with Specific Erosion Parameters

Parametrizing the MB Erosion Model Specifically for the SGM

We apply the properties of the measured SPR for the erosion rates in order to adapt the Montgomery & Brandon (2002) relationship to the SGM. We found $E_0 = 40\text{m Myr}^{-1}$, $S_c = 50^\circ$ (not quite the Heimsath et al. (2012) value of 45°), and $K = 2.3\text{m Myr}^{-1}$. These parameter values are for slopes expressed in degrees. We show the comparison between predicted and observed SPRs in Figure 4. Note that the modeled SPR reaches 170m Myr^{-1} at about 32.5° (close to the suggested cut-off of 30°) and 370m Myr^{-1} at 42° , in general accord with the authors' discussion and assumed S_c of 45° . The value of about 600m Myr^{-1} at 45° is in accord with the only data point (594m Myr^{-1} at 45°) for slopes greater than 40° .

Results for Exponential and Percolation Soil Depth Predictions for the SGM

We show results for the exponential formulation of the soil production function first, and then the power-law formulation. In order to evaluate Eq. (3) for the Heimsath et al. (2012) soil depth, it is necessary to select values of x_s and R_0 . In Heimsath et al. (2012) two different values for both R_0 (170m Myr^{-1} and 370m Myr^{-1}) and x_s are given for the two different slopes, though the two x_s values are similar at about 30cm. Here, no comparable distinction is made for the two slope ranges. Restricting to the single value of $R_{\max} = 506\text{m Myr}^{-1}$, and choosing $x_s = 11.3\text{cm}$, leads to the best fit, with a slope of essentially 1 and an intercept of zero. The value of R^2 is unaffected by the choice of parameters. This value of 506m Myr^{-1} is larger than all the experimentally determined values except one (594m Myr^{-1}), which leads to the prediction of a negative soil depth in that case. However, even the fit parameter 506m Myr^{-1} is larger than the largest value characteristic of the highest slopes (370m Myr^{-1}) reported by

Heimsath et al. (2012). The result of the comparison is shown in Fig. 5a. Note that use of a single SPR function requires a larger value of both the maximum SPR and a more rapid decrease in SPR with depth than are reported in Heimsath et al. (2012), but the discrepancies are not large.

Use of the percolation prediction leads to the comparison in Fig. 5b. Although there are no adjustable parameters, the prediction is, on the average, off by only about 14%, while the R^2 value is noticeably greater than obtained for the exponential phenomenology, 0.59 instead of 0.39. Here, we have for three sites replaced the extremely low measured SPR values by values expected from the MB formulation. We can give three reasons for this; first, the measured SPR values give soil depths that are much too large; secondly, the measured SPR values are much smaller than any others with similar slopes, and third, the regression for the SPR as a function of soil depth given in Heimsath et al. (2012) is not consistent with these three sites either. In the first argument, the three SPR values correspond to erosion rates of 12m Myr⁻¹, 21m/Myr⁻¹ and 10m/Myr⁻¹ for slopes with angles 17°, 0°, and 5°, respectively. The MB phenomenological result with best fit parameters for the SGM yields erosion rates of 84m/Myr⁻¹, 40m Myr⁻¹, and 52m Myr⁻¹, however, for these three slope angles. If the values obtained from the fit of the MB erosion rate to the data are used for the erosion rates at these three sites, then adding these three sites back to the analyzed data is completely consistent with the remaining data points. (The addition of these three points produces only a small change in the R^2 value, from 0.583 to 0.586). Moreover, the Heimsath regression for slopes less than 30°, $R = 170\text{m Myr}^{-1} \exp(-0.031x)$ yields 32m Myr⁻¹ for the deepest soils measured ($x = 54\text{cm}$). This value of 32m Myr⁻¹ is also only 20% smaller than the background denudation rate of 40m Myr⁻¹, which we used to generate the best-fit parameters for the MB phenomenology, but triple the smallest SPR values measured.

The comparisons of theory with experiment may also be put on a single graph (Fig. 6) showing results from both the exponential depth dependence of the SPR and

the percolation theoretical result. The most obvious distinction is the negative curvature of the predicted results from the exponential phenomenology. An alternative representation is to plot both predicted and observed depths for each site as functions of slope angle (Fig. 7). Although the soil depth is represented as a function of slope angle, it is calculated here point-by-point from the observed, rather than modeled, soil production function. The reason that the R^2 value for the percolation treatment (0.59) is higher than that for the exponential model (0.39) is now visible. The advantage is associated with the wider range of predicted soil depths at any given slope using the percolation model on account of its greater sensitivity to the SPR. Thus, the exponential model predictions tend to lie more nearly in the center of the envelope of measured soil depths at a given slope angle.

It is interesting to use the Montgomery and Brandon erosion rate phenomenology, which was tested on the actual SPR data from the SGM, to generate a steady-state soil depth across the SGM sites by both percolation and exponential representations, as well as the percolation result for maximum soil depth attained between landslides. The result is shown in Fig. (8). For most of the range of slope angles, the distinction between the steady-state percolation and exponential phenomenologies is small, but it should be noted that the percolation parameters were determined from other measurements, rather than from an optimal agreement with experiment. If the maximum percolation prediction using the MB slope erosion-rate function and the full time between landslides is indeed reliable, one could infer that the 8 (of a total 55) sites above this curve are susceptible to landslides, except that the authors (Heimsath et al. 2012) noted specifically that they tried to avoid this eventuality by picking sites with rounded topography near ridge crests. Nevertheless, Figure 9. shows the close proximity of two such sites to landslide scars.

Finally, it should be noted that the maximum soil depth from the stochastic solution seems to provide an upper limit for soil depths only at slopes larger than approximately 10-15°, roughly in accord with the authors' observation that such

landsliding is dominant only on slopes greater than 30°. At slope angles smaller than 10°, the upper bound for soil depths is more nearly consistent with the steady-state percolation calculation, a factor 2 smaller.

4.4. Slope-dependent Soil Depth, Eq. (14), Imported Erosion Parameters

Figure 10 shows a comparison of the slope-dependent development of soils in three studies of gradual soil erosion (Moore *et al.*, 1993; Buttle *et al.*, 2004; Norton & Smith (1930) as digitized from Jenny, 1941), as well as one case, the Appenine Mountians (AM) where soil erosion is predominantly by landsliding (Salciarini *et al.*, 2006). The identical function (Eq. 14) with the same erosion parameters as for the Olympic Mountains was used to predict all soil depths. Note that the Norton and Smith data were already considered in Yu *et al.* (2017a). However, in each of the four cases the data were multiplied by a constant factor, representing distinctions in particle size and infiltration rates, in order to generate closest agreement with Eq. (16). We investigated the sensitivity of the comparison to the values of these constants and found that they are accurate within about 5%. The resulting four “fit” prefactors are given in Table 1. Four corresponding parameter values, as estimated from comparison with the particle size and infiltration data of the SGM, are also given in Table 1. Comparing the values pairwise suggests that our theoretically-based estimates are generally within a factor 2 or less of the best-fit values.

What we infer from this comparison is: 1) the soil production model appears to be in accord with these data as well, 2) the erosion model of Montgomery and Brandon (2002) is probably applicable in a wide range of cases, since it is compatible with all the data considered here, although two distinct sets of parameter values needed to be employed, one for the SGM, and a second for the other 4 sites.

5. Conclusions

The soil formation model derived from percolation theory can be used to predict soil recovery after landslides. In its steady-state form, it can also be used to generate an expression for a mean soil depth as a function of slope by adopting an appropriate relationship between erosion rate and slope angle. Addressing the slope dependence of soil depths introduces additional possibilities to test the predictions, but also introduces additional uncertainties in input parameters. At steep slopes, where erosion of soil is mass wasting-dominated or susceptible to shallow landsliding, the steady-state predicted soil depth may be interpreted as a temporal and/or spatial mean soil depth. The predicted soil depth neglecting gradual erosion, but evaluated at landslide recurrence intervals, is a factor 2 larger than the steady-state soil depth prediction for the same total erosion rate. The mean soil depth in threshold landscapes is not greatly different from that where steady-state models are appropriate, as long as the total erosion rate is the same. Using a single published relationship for the erosion rate as a function of slope angle, it is possible to generate a quasi-universal prediction of mean soil depth as a function of slope angle. For both the case where gradual erosion processes dominate and when landsliding dominates erosion, the soil depths predicted thus are often in very close agreement with observed depths. It appears likely that the present formulation of soil depth as a function of erosion rate has the potential to explain slope angle dependences of soil depth as well, provided it is possible for any given geographic region to generate a reliable model of erosion rate as a function of slope angle. Even though Heimsath et al. (2012) explicitly avoided measuring soil production rates on landslide scars at steeper slopes, our predictions regarding stability mapped out rather accurately the maximum soil depth observed as a function of slope angle, while those sites with slightly greater depths were sometimes found adjacent to landslide scars. Thus, a

method to generate a maximum stable soil depth as a function of slope angle may provide a means to distinguish soils susceptible to landsliding.

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Table 1. Particle size and infiltration rate information across sites

Site ^a	SGM	PL	AM	Sterling	N & S
Climate ^b	Mediterranean	Humid continental	Mediterranean	Semi-arid steppe	NA
P (m/yr) ^c	0.63	1.1	0.9	0.4	NA
I (m/yr) ^c	0.145	0.253	0.207	0.09	NA
x_0 (μm)	30	30	1095	30	NA
Factor ^d	1.00	1.90	54.96	0.58	NA
Ratio ^e	NA	1.60	54.96	1.60	1.50
Percentage ^f (%)	NA	15.8	0	175.9	NA

a. SGM = San Gabriel Mountain, PL = Plastic Lake Basin, AM = Apennine Mountains, N & S = data from Norton & Smith (1930).

b. Climate at SGM, PL, AM and Sterling are referenced from Rulli and Rosso (2005), Buttle and House (1997), Salciarini et al. (2006), and Moore et al. (1993).

c. P is precipitation, I is infiltration. Infiltration rate is estimated as 23% of annual precipitation for each site if run-on and run-off values are not available. I value in SGM is referenced from Yu et al. (2017), precipitation in Plastic Lake is 1.1 m yr⁻¹ (Buttle et al., 2004), 0.9 m yr⁻¹ for Apennine Mountains (Salciarini et al., 2006), 0.4 m yr⁻¹ for Sterling, northeastern Colorado (Moore et al., 1993).

d. Factor used to adapt soil depths calculated as: factor = (particle size on site/30) * (infiltration rate on site/0.145)^{1.15}.

e. Ratio necessary to make the data from the 4 sites fit the predicted curve

f. Discrepancy between c) and d) in percent.

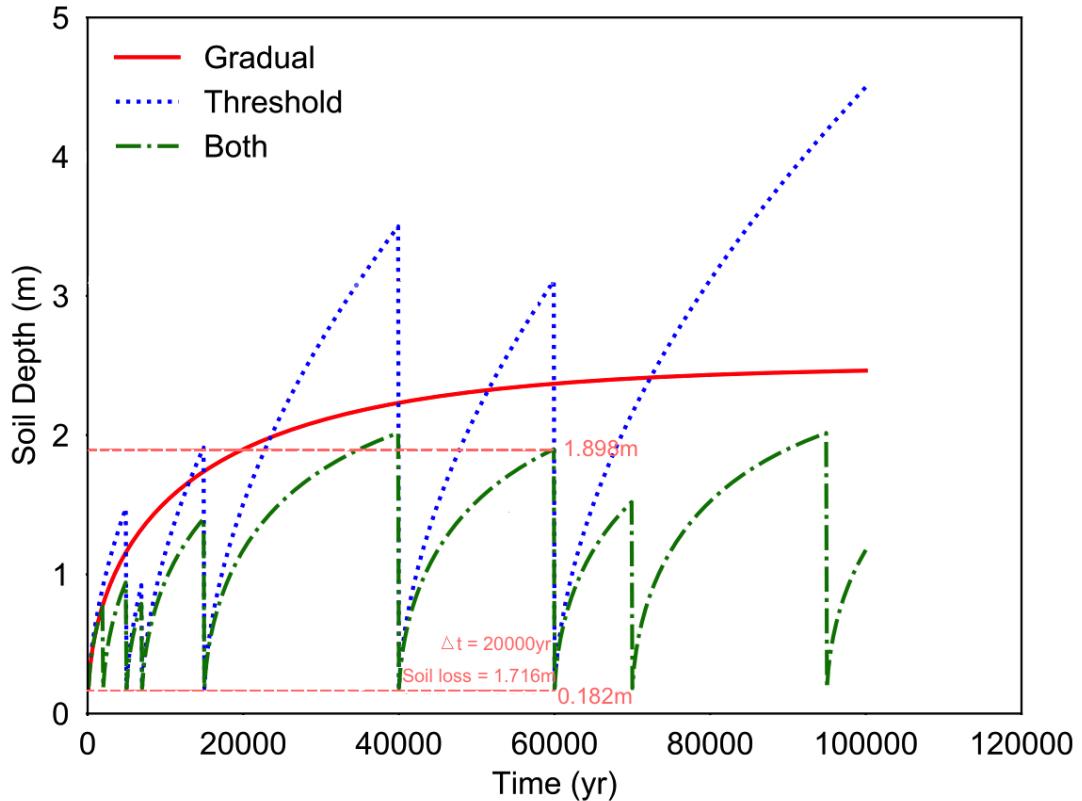


Figure 1. Schematic illustration of soil depths with and without disturbance of shallow landsliding as well as background erosion over time. Red curve includes 100m/Myr soil loss from gradual processes, but no landsliding, blue curve includes 100m/Myr soil loss from landsliding, but no soil loss from gradual processes, while the green curve includes 100m/Myr soil loss from each of the gradual and catastrophic processes. Time of blue curve is generated aperiodically at ages of 5000, 7000, 15000, 40000, and 60000 years. Time of green curve is generated at ages of 2000, 5000, 7000, 15000, 40000, 60000, 70000, and 95000 years.

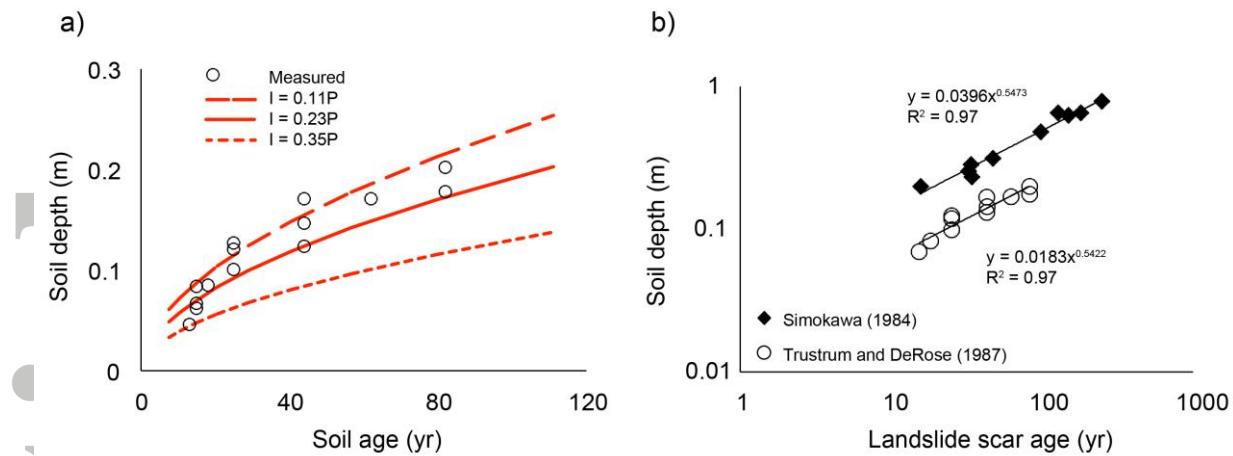


Figure 2. a) Estimated soil depth development in time at the Taranaki site in New Zealand (Trustrum and de Rose, 1988) compared with observed soil depths. Short dashed curve corresponds to $I = 0.11 P$, long dashes to $I = 0.35 P$, and the solid line to $I = 0.23 P$. Other parameters are described in the text. b) Scaling of soil development with age of landslide scars. Data from Shimokawa (1984) is digitized from Iida (1999, Figure 11). First data point at 4 yrs, which is well above the trend line, was omitted. For data from Trustrum and De Rose (1988), first data point at 13yr, well below the trend line, is neglected. Multiple soil depths at 15 years are averaged to reduce scatter.

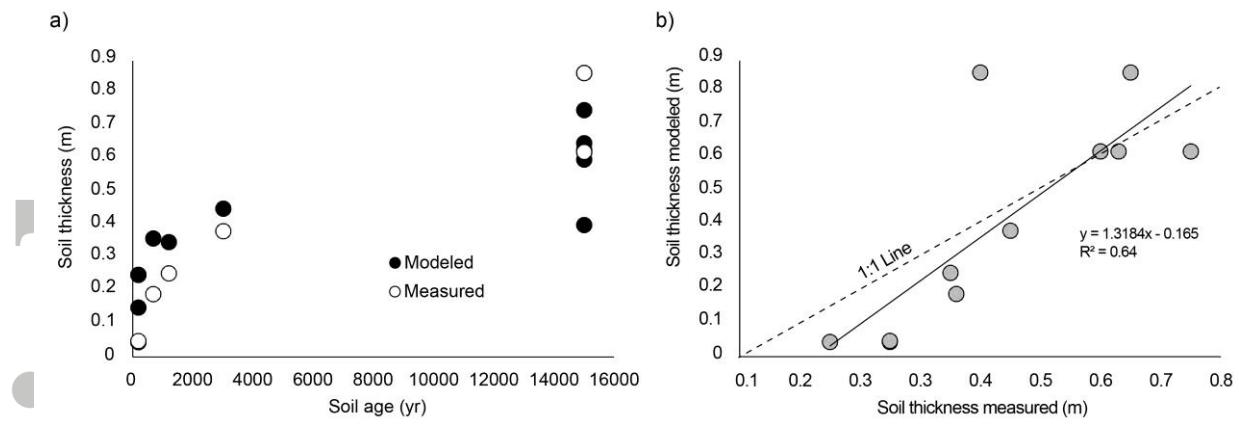


Figure 3. a) Post-landslide soil development at Rossberg, Switzerland as compared with theoretical predictions from Eq. (6). The predictions are made without use of adjustable parameters and include a significant background soil loss from gradual erosion processes, noted particularly at time scales exceeding about 5,000 years. b) Comparison between modeled and measured values. The 1:1 relationship is given by the dashed line.

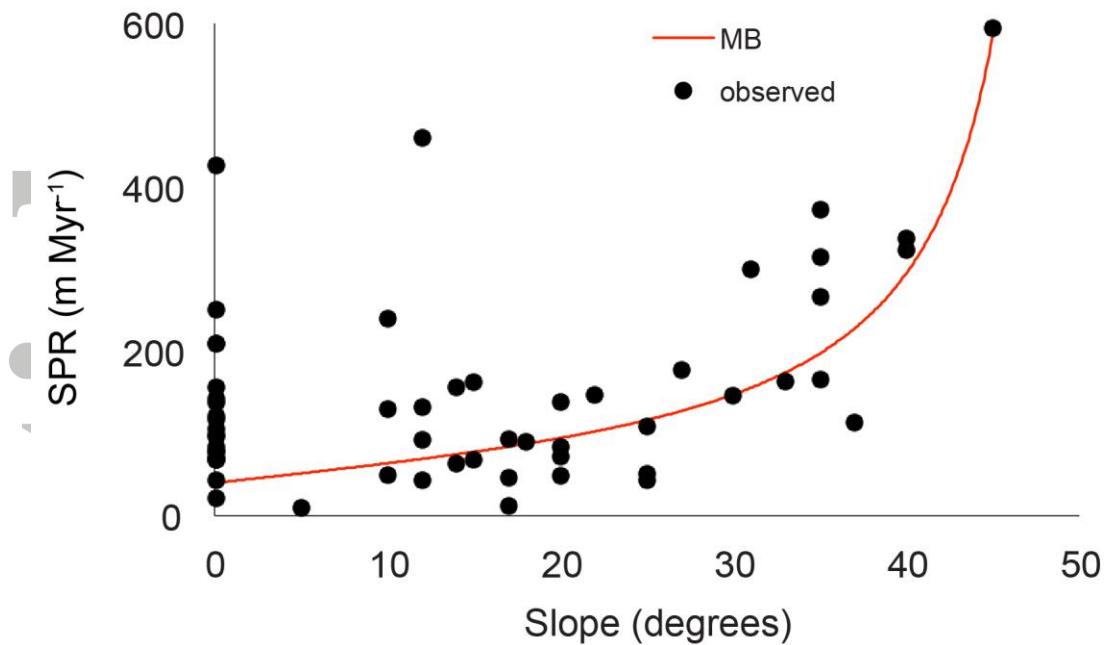
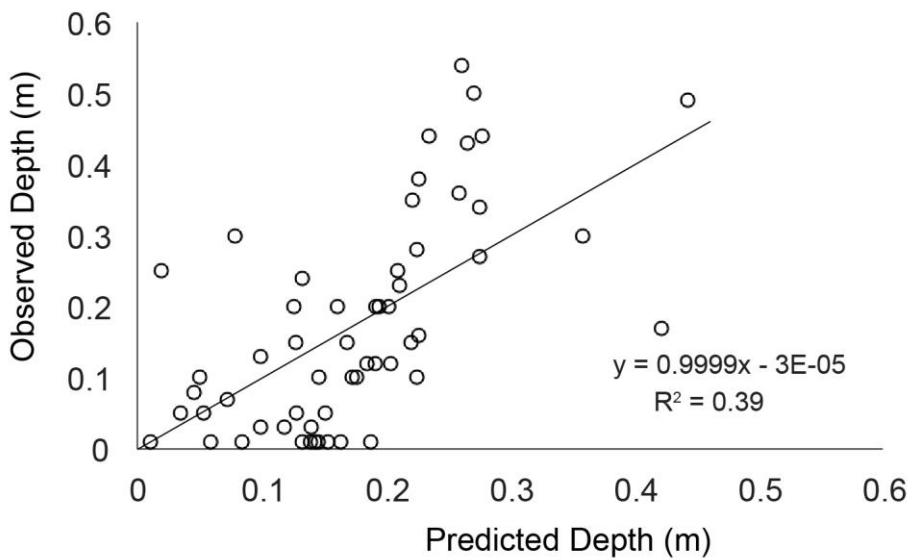


Figure 4. Fit of Montgomery-Brandon (MB) phenomenological relationship for erosion as a function of slope angle to data (Heimsath et al. 2012) for soil production rates as a function of slope angle for the SGM. Parameters are given in the text.

a) Exponential fit



b) Percolation prediction

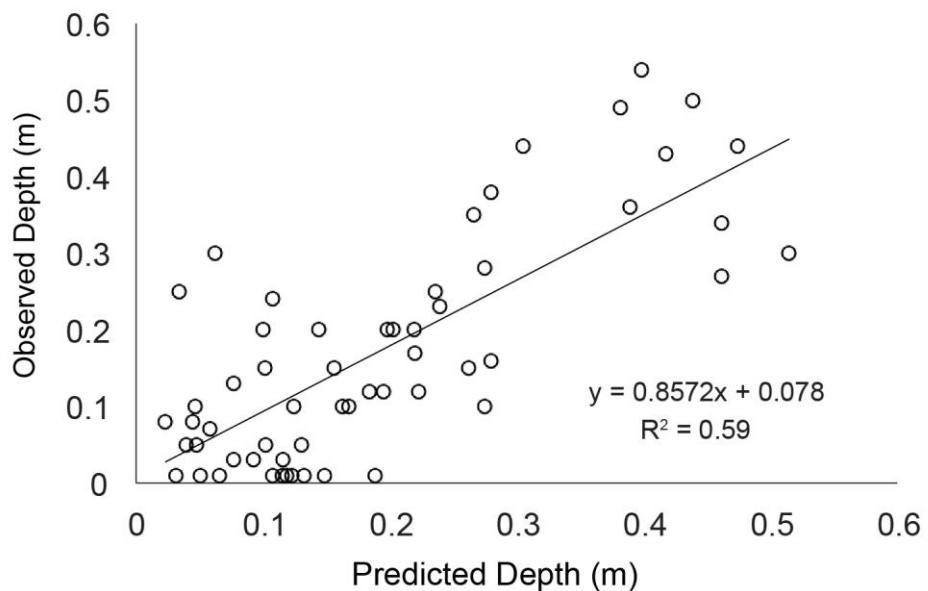


Figure 5. a) Optimized fit for soil depths as a function of soil production rates using exponential phenomenology of Heimsath et al. (2012) and a single maximum soil production rate. Data for the SGM from Heimsath et al. (2012). b) For the same site and data, comparison of percolation predictions for soil depths using observed soil production rates.

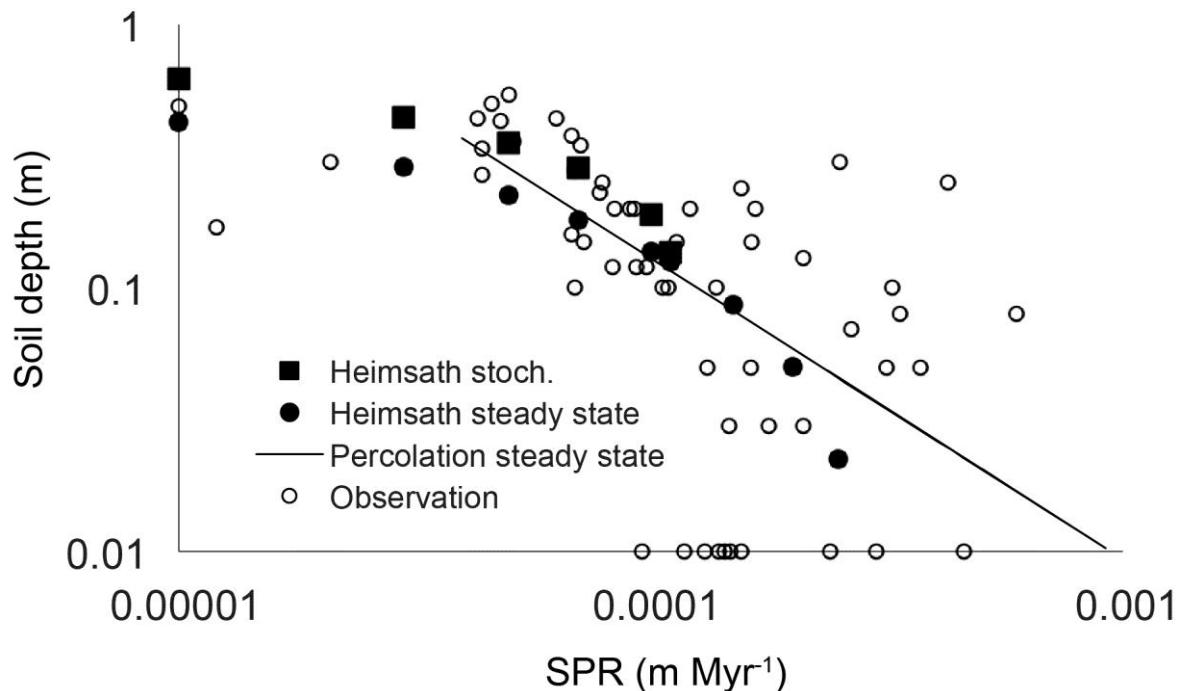
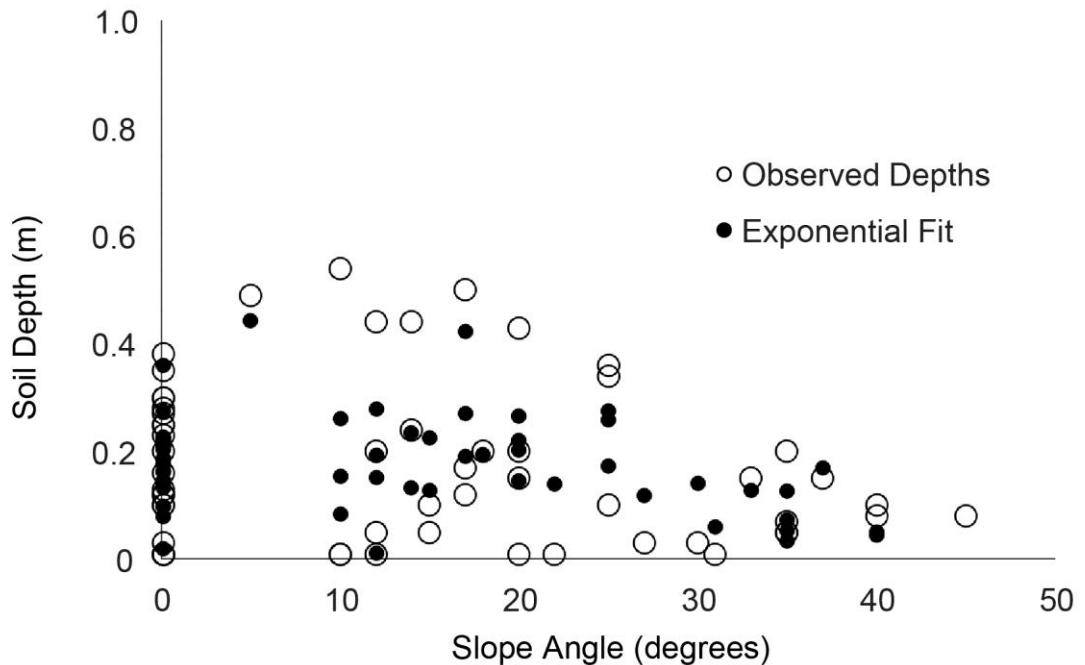


Figure 6. Comparison of exponential phenomenology with percolation prediction and data from the SGM. In this case, the parameters of the exponential phenomenology were chosen in order to generate similar predicted soil depths as obtained from percolation concepts, in order to emphasize the difference in curvature from the two models. The maximum SPR = 300m/Myr and $x_0 = 25\text{cm}$ were applied, in each case within 20% of the values Heimsath et al. (2012) used for steeper slopes. Description of terms: Heimsath stoch.: stochastic means in this context that all erosion is due to landsliding; Eq. 12. Heimsath steady state: according to Eq. 3
Percolation steady state: according to Eq. 16

a) Exponential fit



b) Percolation prediction

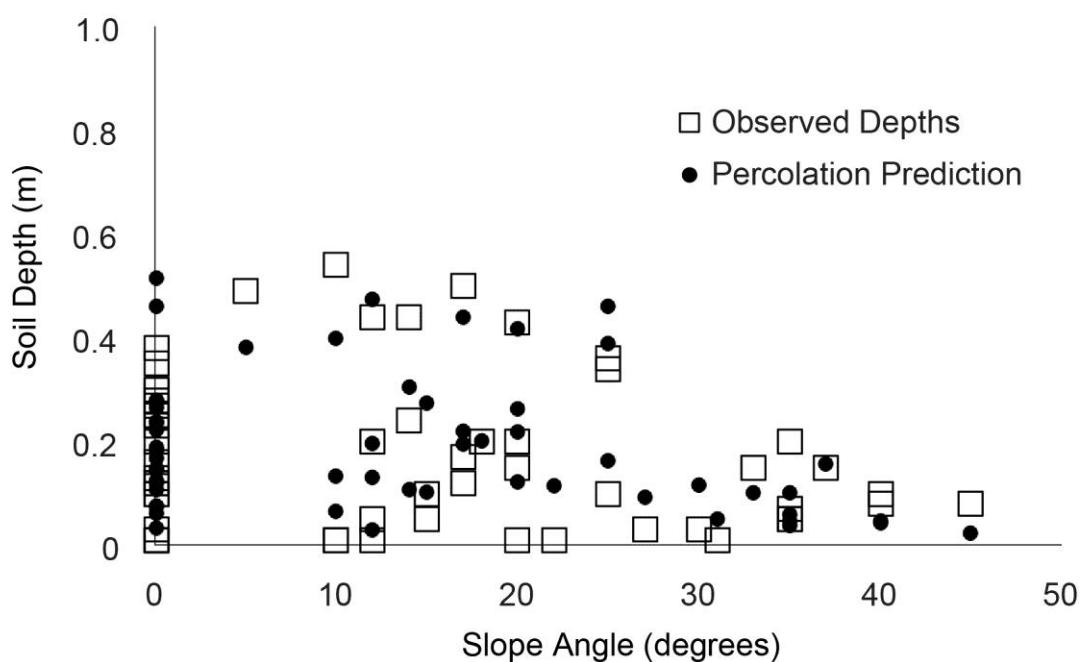


Figure 7. a) Comparison of predicted and observed soil depths as a function of slope angle using the exponential phenomenology and the best fit parameters given in the text. b) Comparison of predicted and observed soil depths using percolation theory for chemical weathering and best estimates for input parameters as discussed in text.

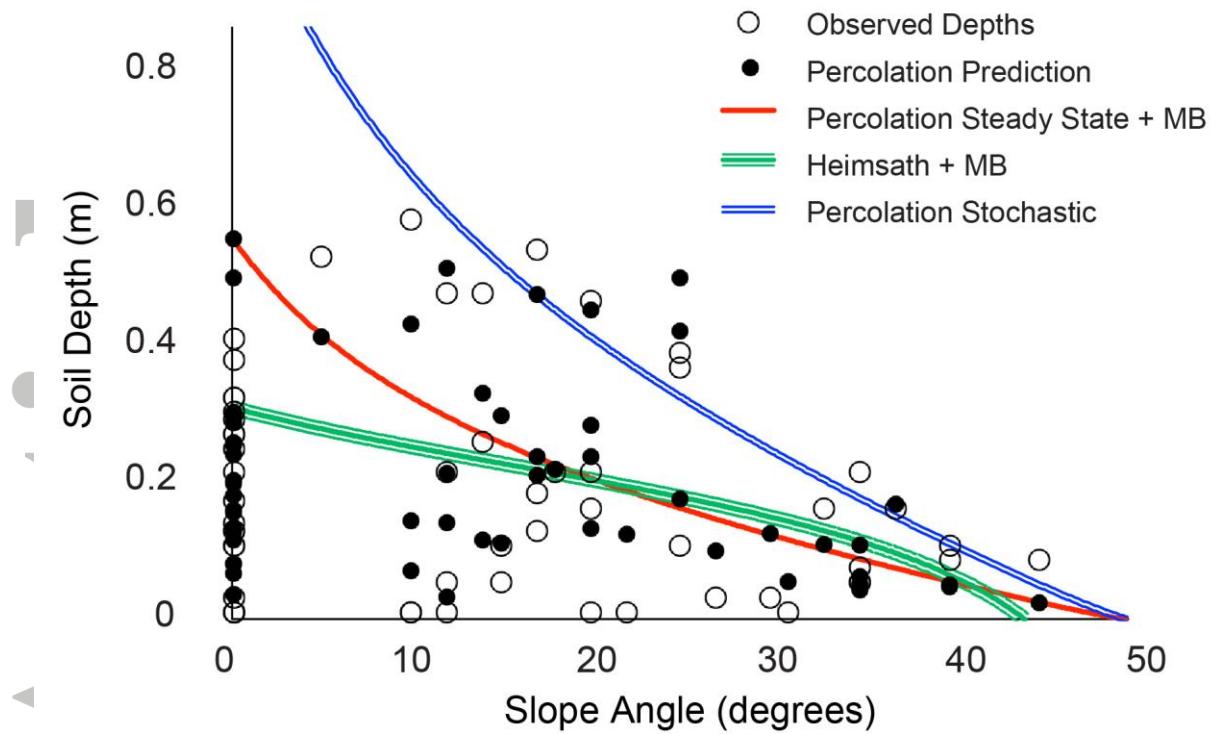


Figure 8. Comparison of exponential fit and percolation prediction for soil depths as a function of slope angle. Description of terms: MB = Montgomery & Brandon (2002), Eq. 15. Percolation prediction: Eq. (6) with measured values of SPR for erosion rates. Percolation steady state: according to Eq. 16. Heimsath: Eq. (3) with MB (Eq. (15) substituted for the denudation rate, D). Percolation Stochastic: Eq. (9) with MB (Eq. 15)

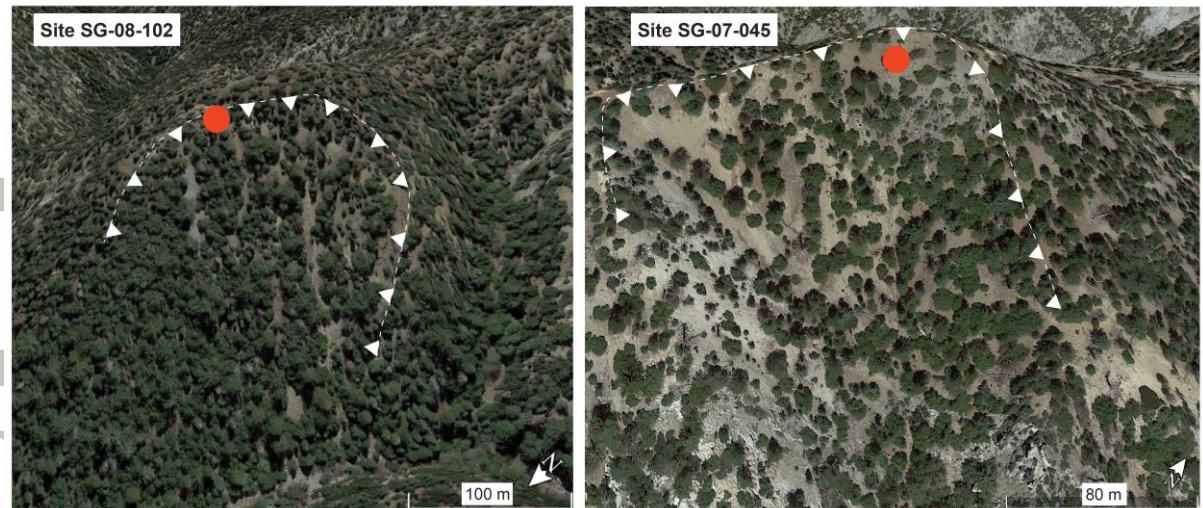


Figure 9. Images of two sites (Heimsath et al., 2012) with actual soil depths exceeding predicted values from stochastic application of percolation theory to maximum stable soil depth in landslide prone areas.

Accepted

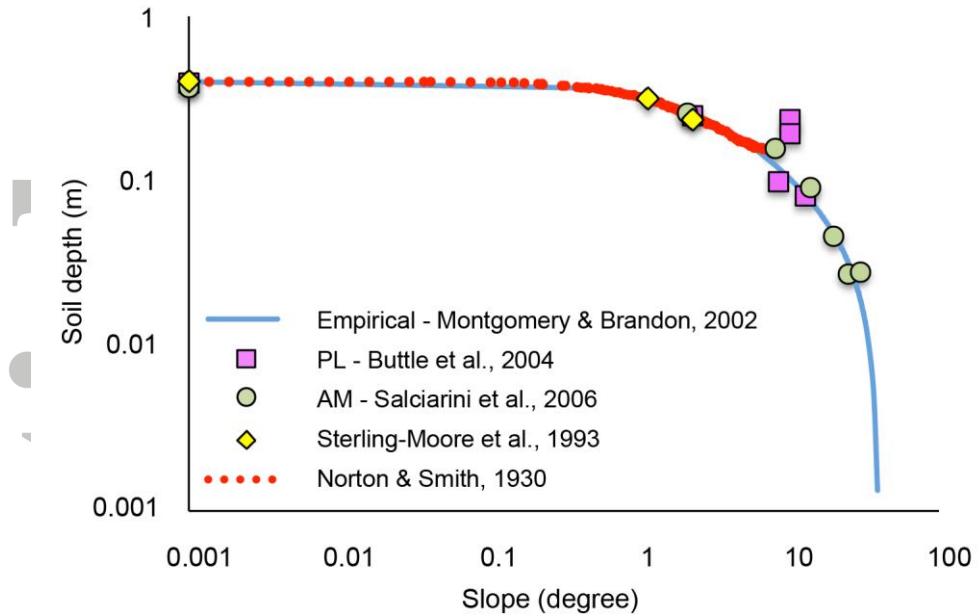


Figure 10. Comparison of predicted soil depth and observed soil depth at 3 study sites exhibiting gradual erosion. PL = Plastic Lake (Buttle *et al.*, 2004), Sterling (Moore *et al.* 1993), and from Norton and Smith (1930) as digitized from Jenny (1941). At one of the four sites, AM = Apennine Mountains from (Salciarini *et al.*, 2006), erosion is chiefly a result of landsliding. Data from AM is adjusted by a factor of 54 due mainly to coarser particle size (see Table 1), but to a lesser extent, also a greater infiltration rate. Constant numerical factor applied to data from remaining sites in order to isolate magnitude and shape of soil depth function (values in Table 1 with discussion in text). Erosion function from Olympic Mountains with original parameters from Montgomery & Brandon (2002) is used for all sites.